$$= \frac{-x^{n+1}[n(x-1)-1]^2 + x(x+1) - x^{n+2}}{(1-x)^3}.$$

Putting $-x$ in place of x, we have

$$S = ax - bx^2 + cx^3 - dx^4 + \ldots.$$

$$= a\left[\frac{x}{1+x}\right] - \Delta a\left[\frac{x}{1+x}\right]^2 + \Delta^2 a\left[\frac{x}{1+x}\right]^3 - \ldots$$

Putting $x = 1$, we have

$$a - b + c - d + e - f + \ldots \text{ ad inf.} = \tfrac{1}{2}a - \tfrac{1}{4}\Delta a + \tfrac{1}{8}\Delta^2 a - \tfrac{1}{16}\Delta^3 a + \ldots.$$

Thus we find, $1-1+1-1+1-1+1-1+\ldots$ ad inf. $= \frac{1}{2}$,

$1-2+3-4+5-6+7-8+\ldots$ ad inf. $= \frac{1}{4}$,

$1-3+5-7+9-11+13-\ldots$ ad inf. $= 0$,

$1-4+9-16+25-36+49-\ldots$ ad inf. $= 0$.

The formula $ax - bx^2 + cx^3 - dx^4 + \ldots$

$$= a\left[\frac{x}{1+x}\right] \quad \Delta a\left[\frac{x}{1+x}\right]^2 + \Delta^2 a\left[\frac{x}{1+x}\right]^3 - \ldots$$

is also very useful, sometimes, to change a series into one that converges more rapidly. For instance,

$$\log(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \ldots.$$

$$= \frac{x}{1+x}\left\{1 + \frac{1}{2}\left[\frac{x}{1+x}\right] + \frac{1}{3}\left[\frac{x}{1+x}\right]^2 + \frac{1}{4}\left[\frac{x}{1+x}\right]^3 + \ldots\right\}.$$

$$\tan^{-1}x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \ldots = \frac{1}{x}(x^2 - \tfrac{1}{3}x^4 + \tfrac{1}{5}x^6 - \tfrac{1}{7}x^8 + \ldots)$$

$$= \frac{x}{1+x^2}\left\{1 + \frac{2}{3}\left[\frac{x^2}{1+x^2}\right] + \frac{2.4}{3.5}\left[\frac{x^2}{1+x^2}\right]^2 + \frac{2.4.6}{3.5.7}\ \&c.\right\}.$$

The above method of the summation of series is by *Euler*. He published it in his Inst. Calc. Diff.

QUERY BY E. HANCOCK, ELGIN, ILL. — If a wheel of 60 teeth work in a pinion of 10 leaves, the distance between their centres being .455 in., the diameters of the pitch circles will be .78 in., $= a$, and .13 in., $= b$, respectively. Allowing one-half for space between the teeth, each tooth will occupy a central angle of 3°, $= c$. Suppose the teeth to terminate at the intersection of two similar epicycloidal curves the diameter of whose generating circle is .065, $= \frac{1}{2}b$; what must be the *addendum* to the wheel outside of the pitch circle, i. e., what is the length of the teeth, outside of the pitch circle?